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*by* Manuharawati Manuharawati

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# Some Convergence Theorems On $\delta$ −Denjoy Integral

Manuharawati
Department of Mathematics, Universitas Negeri Surabaya
Jalan Ketintang, Surabaya 60231, Indonesia

Several descriptive integrals were contructed by using the concept of absolute continuity on an interval $[a,b] \subset \mathbb{R}$. By generalizing interval $[a,b]$ into fundamental $\delta$ −interval for a positive function $\delta$ on interval $[a,b]$, we can construct an absolute $\delta$ −continuous function on interval $[a,b]$. In this paper, the author used literature review to construct a Denjoy integral with respect to function δ which is called δ −Denjoy integral. In addition, the author proved the Cauchy theorem and Harnach theorem on such an integral.

**Keywords:** Cauchy theorem, $\delta$ −Denjoy integral, Harnach theorem.

## 1. INTRODUCTION

There have been many descriptive integrals studied by such mathematicians [2-3, 5, 7-9, 11-12, 15-16].

Using absolute continuity concept which involves interval system [1, 5, 11, 16] there have been studied Denjoy integral on a cell $[a,b] \subset \mathbb{R}$. Schwabik [15] introduces descriptive integral using generalized ordinary differential equation approach while the others, using approximately interval system.

Quite different from the afore mentioned authors, we will study descriptive integral, especially Denjoy integral type that the absolute continuity concept used refers to absolute continuity [13-14], that is, the absolute continuity which involves δ −fundamental interval system [4, 10]. Beside that, we will study the Cauchy theorem and Harnach theorem on this integral later.

## 2. BASIC CONCEPT

In this section, we study some basic concepts and related properties that is useful for later discussion.

**Definition 2.1**[4, 14] *Given two real numbers $a, b$ with $a \neq b$. If $\delta: [a,b] \to \mathbb{R}^+$ and $y \in I$, then an interval $(s,t) = \{u \in [a,b]: y - \delta(y) \leq s < u < t \leq y + \delta(y)\}$ is called a $\delta$ − fundamental interval at $y$. We denote $\mathcal{D}_y$ for a collection of all fundamental $\delta$ −intervals at $y$. Some properties of $\mathcal{D}_x$ are*

D1. *For every $D_y \in \mathcal{D}_x$ and $s < y < t$,*
$$D_y' = D_y \cap (s,t) \in \mathcal{D}_x.$$

D2. *If $D_x'$, $D_x'' \in \mathcal{D}_x$, then*
$$D_x' \cap D_x'' \in \mathcal{D}_x.$$

D3. *Given an index set $U$. If for any $\beta \in U$ we have $D_x{}^{\beta} \in \mathcal{D}_x$, then*
$$D_x = \bigcup_{\beta \in U} D_x{}^{\beta} \in \mathcal{D}_x.$$

D4. *For every $D_l \in \mathcal{D}_x$, $x \in D_l$ and there are $u, v \in D_l$ with*
$$u < x < v.$$

D5. *Let $D_x$ be elemen of $\mathcal{D}_x$. If $u, v \in D_x$ and $u < x < v$, then there exists $m, n \in D_x$ with*
$$u < m < x < n < v.$$

*Email Address: manuhara1@yahoo.co.id

Note that $\mathcal{D}_x \neq \emptyset$. So, we can take exactly one $D_x \in \mathcal{D}_x$. The collection of all $D_x$ such as denoted by $\mathcal{G}_x$.

**Definition 2.1**[4] *A collection $\Delta$ of all closed intervals $[u,v]$ with $u,v \in D_x \in \mathcal{G}$ is called a $\Delta$ –fundamental full cover ($FFC-\Delta$) on $[a,b]$ and $\mathcal{G}$ is called a generator of $FFC-\Delta$.*

**Definition 2.2**[10] *Given two real numbers $a,b$ with $a \neq b$, $\delta: [a,b] \to \mathbb{R}^+$, and $E \subset \mathbb{R}$. The ral enumber $l$ is said to be $\delta$ –limit point of a set $E$ if for any fundamental $\delta$ –interval $D_l \in \mathcal{D}_l$ we have*

$$D_l \cap E - \{l\} \neq \emptyset.$$

**Definition 2.3**[10] *Given two real numbers $a,b$ with $a \neq b, \delta: I = [a,b] \to \mathbb{R}^+$. A real function $H$ defined on $I$, and $c$ be $\delta$ –limit point of $I$. The real number $l$ is called to be $\delta$ –limit of a function $H$ for $x$ closed to $c$ and denoted by*

$$\delta - \lim_{x \to c} H(x) = l$$

*if for every $\alpha \in \mathbb{R}$, $\alpha > 0$, there exists a fundamental $\delta$ –interval $D_c \in \mathcal{D}_c$ such that for eny $y \in D_c, y \neq c$ implies*

$$|H(y) - l| < \alpha.$$

**Definition 2.4**[10] *Given two real numbers $a,b$ with $a \neq b$ and $\delta: I = [a,b] \to \mathbb{R}^+$. If a real function $H$ on $I$, $c$ be $\delta$ –limit point of $I$ and*

$$\delta - \lim_{x \to c} H(x) = H(c)$$

then $H$ *is said to be $\delta$ –continuous at $c$*

**Definition 2.5**[10] *Given two real numbers $a,b$ with $a \neq b$, $\delta: I = [a,b] \to \mathbb{R}^+$ and $c \in [a,b]$. If the real number*

$$D_\delta H(c) = \delta - \lim_{y \to c} \frac{H(y) - H(c)}{y - c}$$

*exists, then $D_\delta H(c)$ is called to be $\delta$ –Derivative of a function $H: I \to \mathbb{R}$ at $c$.*

If two real numbers $a,b$ with $a < b$ be given and $\delta: [a,b] \to \mathbb{R}^+$ We have some types of $\delta$ –absolute continuity af a function.

**Definition 2.6**[13] *Given two real numbers $a,b$ with $a < b$, $\delta: I = [a,b] \to \mathbb{R}^+$, $Y$ be a subset of $\mathbb{R}$ and $H$ be a real function on $I$.*

(i) *If for every $\varepsilon \in \mathbb{R}, \varepsilon > 0$, there exists $\rho \in \mathbb{R}^+$, such that for every sequence $(x_i)$ on $Y$ there is $(D_{x_i})$, $D_{x_i}$ are nonoverlapping, $D_{x_i} \in \mathcal{D}_{x_i}$ such that if $\sum_i |x_i - u_i| < \rho$, $u_i \in D_{x_i}$ implies*

$$\sum_i |H(x_i) - H(u_i)| < \varepsilon.$$

*then a function $H$ is said to be weakly absolutely $\delta$ – continuous on $Y$. We denote $AC_\delta(Y)$ for the set of all $H$ with $H$ is weakly absolutely $\delta$ – continuous functions on a subset $Y$ of $[a,b]$.*

(ii) *If we have $Y_1, Y_2, Y_3, \ldots$ with properties $Y = \bigcup_i Y_i$ and $H \in AC_\delta(Y_i)$ for every $i$, then $H$ is said to be generalized weakly absolutely $\delta$ –continuous on $Y$. We denote $ACG_\delta(Y)$ for the set of all $H$ with $H$ is generalized weakly absolutely $\delta$ –continuous functions on a $Y$.*

(iii) *If for every $\varepsilon \in \mathbb{R}, \varepsilon > 0$, there exists $\rho \in \mathbb{R}^+$, with properties: for every $x_1, x_2, x_3, \ldots$ $x_i \in Y$ we have $(D_{y_i})$, $D_{y_i}$ are nonoverlapping $D_{y_i} \in \mathcal{D}_{y_i}$ such that $\sum_i |y - u_i| < \rho$, $u_i \in D_{y_i}$ implies*

$$\sum_i \omega(H; [a_i, b_i]) < \varepsilon$$

*with $a_i = min\{y_i, u_i\}$ and $b_i = max\{y, u_i\}$ then $H$ is said to be strongly absolutely $\delta$ – continuous on $Y$. We denote $AC_\delta^*(Y)$ for the set of all $H$ with $H$ is strongly absolutely $\delta$ –continuous functions on a set $Y$.*

(iv) *If we have $Y_1, Y_2, Y_3, \ldots$ with properties $Y = \bigcup_i Y_i$ and $H \in AC_\delta^*(Y_i)$ for every $i$, $H$ is said to be generalized strongly absolutely $\delta$ –continuous on a set $Y$ and $ACG_\delta^*(Y)$ denote the set of all $H$ with $H$ is generalized strongly absolutely $\delta$ –continuous functions on a set $Y$.*

The following theorems will be used in next discussion.

**Theorem 2.1**[13] *Given a positive real function $\delta$ on closed and bounded interval $I = [a,b]$. The $C_\delta(I)$, $AC_\delta^*(I)$, and $ACG_\delta^*(I)$ are linear spaces.*

**Theorem 2.2**[13] *Given a positive real function $\delta$ on closed and bounded interval $I = [a,b]$, $H$ be a real function on $I$ and $s \in I$. If $D_\delta H(s)$ exists, then $H$ is $\delta$ –continuous at a point $s$.*

**Theorem 2.3**[14] *Given two real numbers $a,b$ with $a < b$. If $D_\delta H(s)$ exists for every $s$ on $I = [a,b]$, then $H \in ACG_\delta^*(I)$.*

**Theorem 2.4**[14] *Given two real numbers $a,b$ with $a < b$ If $I = [a,b]$ and $D_\delta H(s)$ exists for all $s$ nearly everywhere on interval $I$, then $H \in ACG_\delta^*(I)$.*

## 3. RESULT AND DISCUSSION

**Definition 3.1:** *Given two real numbers* $a,b$ *with* $a<b$, *and* $\delta: I=[a,b]\rightarrow \mathbb{R}^{+}$. *If there exists a real function* $H$ *on* $I$ *with properties*:

(i) $D_{\delta}H(s)=h(s)$ *for all* $s$ *allmost everywhere on* $[a,b]$,

(ii) $H\in C_{\delta}(I)\cap ACG_{\delta}^{*}(I)$,

*then a real function* $h$ *defined on* $I$ *is said to be* $\delta-$*Denjoy integrable or* $D_{\delta}^{*}-$*integrable on* $I$. *The real number*

$$(D_{\delta}^{*})\int_{a}^{b} h(s)ds=H(b)-\ H(a)$$

*is called to be* $D_{\delta}^{*}-$ *integral value of a function* $h$ *on* $I$ *and a function* $H$ *is called* $D_{\delta}^{*}-$*primitive of* $h$ *on* $I$. *We denote* $D_{\delta}^{*}(I)$ *for the set of all* $h$, *with* $h$ *is a* $D_{\delta}^{*}-$*integrable functions on* $I$..

It is easy to understand that the $D_{\delta}^{*}-$integral value of a function $h$ on $I=[a,b]$ is unique.

**Theorem 3.1:** *If* $s,h\in D_{\delta}^{*}[a,b]$ *and* $\beta$ *is a real number, then* $s+h\in D_{\delta}^{*}[a,b]$ *and* $\beta s\in D_{\delta}^{*}[a,b]$.

**Proof:** Let $S$ and $H$ be $D_{\delta}^{*}-$primitive of $s$ and $h$ on $[a,b]=I$. Then $D_{\delta}H(y)=h(y)$ allmost everywhere. on $I$ and $S,H\in C_{\delta}(I)\cap ACG_{\delta}^{*}(I)$. By applying Definition 2.5, Theorem 2.1 and Theorem 2.2, $D_{\delta}^{*}[a,b]$ is a linear space, i.e.: $s+h\in D_{\delta}^{*}[a,b]$ and $\beta s\in D_{\delta}^{*}[a,b]$. ■

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**Theorem 3.2:** *If* $h\in D_{\delta}^{*}[a,b]$ *and* $[s,t]\subset[a,b]$, *then* $h\in D_{\delta}^{*}[s,t]$.

**Proof:** Let $H$ be a $D_{\delta}^{*}-$primitive function of $h$ on $I=[a,b]$. By Definition 3.1, we have $H:[a,b]\rightarrow\mathbb{R}$ with $D_{\delta}H(y)=h(y)$ *allmost everywhere on* $I$ and $H\in C_{\delta}(I)\cap ACG_{\delta}^{*}(I)$. Since $[s,t]\subset[a,b]$, then, $H:[s,t]\rightarrow\mathbb{R}$ with $D_{\delta}H(y)=h(y)$ allmost everywhere on $[s,t]$ and $F\in C_{\delta}[s,t]\cap ACG_{\delta}^{*}[s,t]$. Since $H\in ACG_{\delta}^{*}[a,b]$ then by applying Definition 2.5, there are $Y_1,Y_2,Y_3,\ldots$ with properties $\bigcup_{i\in\mathbb{N}}Y_i=I$ and $F\in AC_{\delta}^{*}(Y_i)$ for eny $i\in\mathbb{N}$. Define

$$L_i=Y_i\cap[u,v].$$

Then we have: $[u,v]=\bigcup_{i\in\mathbb{N}}L_i$ and $H\in AC_{\delta}^{*}(L)$ for every $i$. So, $H\in ACG_{\delta}^{*}[s,t]$. Consequently $H\in D_{\delta}^{*}[s,t]$. ■

**Theorem 3.3:** *Given two real numbers* $c,d$ with $c<d$ *and* $I=[c,d]$. *If* $s\in D_{\delta}^{*}(I)$ *and* $p$ *be an interior poin of* $I$, *then*

$$(D_{\delta}^{*})\int_{c}^{d} sdt=(D_{\delta}^{*})\int_{c}^{p} s(x)dx+(D_{\delta}^{*})\int_{p}^{d} s(x)dx.$$

**Proof:** By using Theorem 3.1, $s\in D_{\delta}^{*}[c,p]$ and $s\in D_{\delta}^{*}[p,d]$. If $S$ is a $D_{\delta}^{*}-$primitive of $s$ on $[c,d]$, then

$(D_{\delta}^{*})\int_{c}^{d} sdt=S(d)-S(c)$

$=[S(d)-S(p)]+[S(p)-S(c)]$

$=(D_{\delta}^{*})\int_{c}^{p} s(x)dx+(D_{\delta}^{*})\int_{p}^{d} s(x)dx.$ ■

**Theorem 3.8** (Cauchy Theorem): *If* $s\in D_{\delta}^{*}[p,q]$ *for every* $[p,q]\subset[c,d]$ *with* $c<p<q<d$ *and*

$$\delta-\lim_{p\rightarrow c^{+};q\rightarrow d^{-}}(D_{\delta}^{*})\int_{c}^{d} s(t)dt=A\ (exist),$$

*then* $s\in D_{\delta}^{*}[c,d]$ *with*

$$(D_{\delta}^{*})\int_{c}^{d} s(t)dt=A.$$

**Proof:** By applying Theorem 3.2, we have a function $S:[c,d]\rightarrow\mathbb{R}$ with $S(x)=(D_{\delta}^{*})\int_{c}^{x} s(t)dt$ for every $x\in[c,d]$. So, we have $D_{\delta}S(x)=s(x)$ allmost everywhere on $[c,t]$ and $H\in C_{\delta}[c,t]\cap ACG_{\delta}^{*}[c,t]$ for every $\in[c,d]$.
Since

$$\delta-\lim_{x\rightarrow d^{-}}S(x)=$$
$$\delta-\lim_{;x\rightarrow d^{-}}(D_{\delta}^{*})\int_{c}^{d} s(t)dt=A,$$

then for eny real number $\varepsilon>0$ there exists a $\delta-$fundamental interval $D_d\in\mathcal{D}_d$ such that for every $u\in D_d$ with $u<d$ we obtain

$$|S(u)-\ A|<\frac{\varepsilon}{2^{k+2}}$$

for every positive integer $k$.
Furthermore, define $S(d)=A$. By the Definition 2.5 and 2.6, we have: $S$ is $\delta-$continuous at $d$ and

$$|S(u)-\ S(d)|<\frac{\varepsilon}{2^{k+2}}$$

for every positive integer $k$.
Hence

$$\omega(S;[u,d])<\frac{\varepsilon}{2^{k+2}}. \tag{3.1}$$

Since $t\in[c,d]$, then we choose $t=u$ such that (3.1) holds. Since $S\in ACG_{\delta}^{*}[c,u]$, then there are $X_1,X_2,\ldots$ of sets with $[c,u]=\bigcup_{i\in\mathbb{N}}X_i$ and for every $i$, we have $S\in AC_{\delta}^{"}(X_i)$. So, there is a positive integer number $m$ such that $u\in X_m$ and $S\in AC_{\delta}^{*}(X_m)$. Consequently, there is a positive real number $\rho$ with properties: if $(x_i)$ is anysequence with $x_i\in X_m$ there are $D_{x_1},D_{x_2},D_{x_3},\ldots$, $\{D_{x_i},i=1,2,3,..\}$ is nonoverlapping, with $D_{x_i}\in\mathcal{D}_{x_i}$ such that for every $u_i \in D_{x_i} \cap X_m$ with $\sum_{i\in\mathbb{N}}|x_i - u_i| < \rho$ we have

$$\sum_{i\in\mathbb{N}} \omega(S;[a_i,b_i]) < \frac{\varepsilon}{2^{k+1}} \qquad (3.2)$$

with $c_i = \min\{x_i,u_i\}$ and $d_i = \max\{x_i,u_i\}$. Furthermore, define a set $Y = X_m \cup [u,b]$. If $(L_i)$ is any sequence on $Y$, then we have a sequence $(D_{x_i})$ of nonoverlapping $\delta -$ fundamental intervals with $D_{x_i} \in \mathcal{D}_{x_i}$. By (3.1) and (3.2), if $u_i \in D_{x_i} \cap Y$ and $\sum_{i\in\mathbb{N}}|x_i - u_i| < \rho$ then

$$\sum_{i\in\mathbb{N}} \omega(S;\ [c_i,d])$$
$$\leq \sum_{i\in\mathbb{N},x_i\in X_m} \omega(S;[c_i,d_i\ ]) + \sum_{i\in\mathbb{N},x_i\in(u,b]} \omega(S;[c_i,d_i\ ]) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

with $c_i = \min\{x_i,u_i\}$ and $d_i = \max\{x_i,u_i\}$. So, $S \in AC_\delta^*(Y)$. Consequently, $S \in ACG_\delta^*[c,d]$.

Since a function $S$ is $\delta$ –continuous at $d$, then we have

$$\delta - \lim_{x\to d} S(x) = \delta - \lim_{x\to d^-} S(x) = A.$$

So, we have

$$(D_\delta^*)\int_a^b s(t)dt = A. \ \blacksquare$$

**Theorem 4** (Harnach Theorem): *Given two real numbers* $c,d$ *with* $c < d$ *and* $I = [c,d]$. *If the following conditions are hold*

(i) *$Y$ be a closed subset of $I$*

(ii) *$([s_i,t_i])$ be a sequence of contiguous closed intervals of $Y$ on $I$,*

(iii) *$I_Y$ is the a charachteristic function on $Y$ ,*

(iv) *$hI_Y \in D_\delta^*(I), h \in D_\delta^*[s_i,t_i]$ with*

$$\sum_{i\in\mathbb{N}}(D_\delta^*)\int_{s_i}^{t_i} hdt < \infty,$$

*then $h \in D_\delta^*[c,d]$ with*

$$(D_\delta^*)\int_c^d h(l)dl = \sum_{i\in\mathbb{N}}(D_\delta^*)\int_{s_i}^{t_i} h(l)dl + \sum_{i\in\mathbb{N}}\left[(D_\delta^*)\int_{s_i}^{t_i} h(l)dl\right].$$

**Proof:** Since $hI_E \in D_\delta^*(I)$ then we have a function $H: I \to \mathbb{R}$ with $D_\delta H(y) = hI_E(y)$ allmost everywhere on $I$ and $H \in C_\delta(I) \cap ACG_\delta^*(I)$. Since $h \in D_\delta^*[s_i,t_i]$, then for all positive integer number $i$ we have a function $H_i: [s_i,t_i] \to \mathbb{R}$ with $H_i \in C_\delta[s_i,t_i] \cap ACG_\delta^*[s_i,t_i]$. Since $[s_i,t_i]$ is closed and contiguous of $E$ on $[c,d]$, then we have

$$[c,d] - Y = \bigcup_{i\in\mathbb{N}}[s_i,t_i].$$

Furthermore, if we define a real function $G$ on $[c,d]$ where

$$G(y) = \begin{cases} H(y) + \sum_{i=1}^{k-1} H_i(y); y = s_k\ ;\ s_k = t_{k-1}\ or\ s_k \neq t_{k-1} \\ H(y);\ y \in Y\ and\ there\ is\ no\ k\ with\ y \geq s_k \\ H(y) + \sum_{i=1}^{k-1} H_i(y) + H_k(y);\ y \in [s_k,t_k], \end{cases}$$

then we have:

(1) $H(y) = h(y)$ allmost everywhere on $[c,d]$.

(2) $H \in AC_\delta^*(Y) \cap ACG_\delta^*[c,d]$.

(3) Since $H \in C_\delta[s_i,t_i] \cap ACG_\delta^*[s_i,t_i]$ then $H \in C_\delta(\bigcup_{i\in\mathbb{N}}[s_i,t_i]) \cap ACG_\delta^*(\bigcup_{i\in\mathbb{N}}[s_i,t_i])$, so $H \in C_\delta^*[c,d] \cap ACG_\delta^*[c,d]$.

By using (1) and (3), we have $h \in D_\delta^*[c,d]$.

Note that

$$(D_\delta^*)\int_c^d (hI_Y)(l)dl = (D_\delta^*)\int_Y h(l)dl.$$

Since $\sum_{i\in\mathbb{N}}(D_\delta^*)\int_{s_i}^{t_i} h(l)dl < \infty$, we obtain

$$(D_\delta^*)\int_a^b h(l)dl = (D_\delta^*)\int_E h(l)dl + (D_\delta^*)\int_{\bigcup_{i\in\mathbb{N}}[s_i,t_i]} h(l)dl$$
$$= (D_\delta^*)\int_a^b (hI_Y)(l)dl + \sum_{i\in\mathbb{N}}\left[(D_\delta^*)\int_{s_i}^{t_i} h(l)dl\right] \blacksquare.$$

## 4. CONCLUSIONS

Given a positive function $\delta$ on cell $[a,b]$. Based on the concept of $\delta$ –fundamental continuity and generalized $\delta -$ fundamental continuity, we have constructed descriptive-type integral which was defined as $\delta -$ Denjoy integral and also successfully proved the Cauchy theorem and the Harnach theorem on the integral.

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